

Dimensional Reduction, Gauged $D = 5$ Supergravity and Brane Solutions

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Abstract

The $U(1)$ gauged version of the Strominger-Vafa five dimensional $N = 2$ supergravity with one vector multiplet is obtained via dimensional reduction from the $N = 1$ ten dimensional supergravity. Using such explicit relation between the gauged supergravity theory and ten dimensional supergravity, all known solutions of the five dimensional theory can be lifted up to ten-dimensions. The eleven dimensional solutions can also be obtained by lifting the ten-dimensional solutions.

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1 Introduction

Recently, there has been renewed activities in the study of gauged supergravity in various dimensions as well as in their solutions. This, to a large extent, is motivated by the conjectured equivalence between string theory on anti-de Sitter (AdS) spaces (times some compact manifold) and certain superconformal gauge theories living on the boundary of AdS [1]. The theories of extended supergravity in various dimensions possess rigid symmetries. A subgroup of these symmetries can be gauged by the vector fields present in the ungauged theory. Gauged supergravity theories exist in space-time dimensions where supersymmetry allows the existence of a cosmological constant. In $D = 11$, $D = 10$ and $D = 9$, a cosmological constant is not possible.

In five dimensions, $N = 2$ supergravity theories can be obtained by gauging the $U(1)$ subgroup of the $SU(2)$ automorphism group of the $N = 2$ supersymmetry algebra, thus breaking $SU(2)$ down to the $U(1)$ group. The $U(1)$ gauge field introduced to gauge the theory can be taken as a linear combination of the abelian vector fields of the ungauged theory with a coupling constant g . The additional couplings of the fermi-fields of the ungauged theory to the $U(1)$ gauge vector field breaks supersymmetry which necessitates the addition of g -dependent gauge invariant terms in order to restore $N = 2$ supersymmetry. The purely bosonic terms added produce the scalar potential of the theory [2].

Gauged supergravity can be obtained by compactifying higher dimensional supergravity on a group manifold. It is usually difficult to find a consistent ansatz for the compactification¹. In particular it was shown recently [4] that the Freedman-Schwarz [5] gauged $N = 4$ supergravity can be obtained by compactifying ten dimensional supergravity on the $SU(2) \times SU(2)$ group manifold. The difficulty arises because one has to identify the vector fields coming from the compactification of the metric with the vector fields coming from the antisymmetric tensor. The vector fields coming from compactifying the metric on a group manifold behave properly as $SU(2) \times SU(2)$ gauge fields. The components of the antisymmetric tensor do not usually behave like $SU(2) \times SU(2)$ gauge fields, and for this to happen a very precise

¹see [3] and references therein.

form for the ansatz of the antisymmetric tensor must be taken. This prescription also works in obtaining $D = 7$ gauged supergravity by compactifying ten dimensional supergravity on $SU(2)$ group manifold as was recently shown in [6]. In a related matter, there are now many known black hole solutions for gauged $D = 5$ supergravity [7, 8, 9, 10]. These solutions could be promoted to solutions of ten and eleven dimensional supergravity if one can embed the five dimensional supergravity in the higher dimensional ones.

It is our purpose in this paper to show that by compactifying and truncating $D = 10$ supergravity to $D = 5$ on an $SU(2) \times U(1)^2$ group manifold one obtains a gauged $D = 5$ supergravity theory with one vector multiplet. The solutions for this model can then be lifted to seven, ten and eleven dimensions. This work is organized as follow. In section two, it is shown how to reduce $D = 10$ supergravity to a particular gauged $N=2$ five dimensional supergravity theory coupled to one vector multiplet. The gauged theory obtained is the $U(1)$ gauged version of the model introduced by Strominger and Vafa[11]. The five dimensional model obtained is then reformulated in the framework of very special geometry in section three. Some particular solutions of the five dimensional theory, as examples, are lifted to seven, ten and eleven dimensions in section four. Finally our results are summarized and discussed.

2 A $D = 5$ Gauged Supergravity From $D = 10$ Supergravity

In this section we consider the dimensional reduction of $D = 10$ supergravity down to $N = 2$, $U(1)$ gauged $D = 5$ supergravity coupled to one vector multiplet. First, the bosonic part of $N = 1$ supergravity action in ten dimensions is

$$\begin{aligned} S_{10} &= \int \hat{e} \left(-\frac{1}{4} \hat{R} + \frac{1}{2} \partial_M \hat{\phi} \partial^M \hat{\phi} + \frac{1}{12} e^{-2\hat{\phi}} \hat{H}_{MNP} \hat{H}^{MNP} \right) d^4x d^6z \\ &\equiv S_{\hat{G}} + S_{\hat{\phi}} + S_{\hat{H}}. \end{aligned} \tag{1}$$

The notation used in this paper is as follows. We denote ten-dimensional quantities by hatted symbols. Base and tangent space indices are denoted by late and early capital Latin letters, respectively. For the four-dimensional space-time, we use late and early Greek letters, respectively, to denote base

space and tangent space indices. Similarly, the internal base space and tangent space indices are denoted by late and early Latin letters, respectively.

$$\{M\} = \{\mu = 0, \dots, 3; m = 1, \dots, 6\}, \quad \{A\} = \{\alpha = 0, \dots, 3; a = 1, \dots, 6\}. \quad (2)$$

The general coordinates \hat{x}^M consist of spacetime coordinates x^μ and internal coordinates z^m . The flat Lorentz metric of the tangent space is chosen to be $(+, -, \dots, -)$ with the internal dimensions all spacelike. Thus the metric is related to the vielbein by

$$\hat{\mathbf{g}}_{MN} = \hat{\eta}_{AB} \hat{e}^A_M \hat{e}^B_N = \eta_{\alpha\beta} \hat{e}^\alpha_M \hat{e}^\beta_N - \delta_{ab} \hat{e}^a_M \hat{e}^b_N, \quad (3)$$

and the antisymmetric tensor field strength is

$$\hat{H}_{MNP} = \partial_M \hat{B}_{NP} + \partial_N \hat{B}_{PM} + \partial_P \hat{B}_{MN}. \quad (4)$$

The coordinates z^m span the internal compact group space. Thus we introduce the functions $U_m^a(z)$ which satisfy the condition [12]

$$\left(U^{-1}\right)_b^m \left(U^{-1}\right)_c^n (\partial_m U_n^a - \partial_n U_m^a) = \frac{f_{abc}}{\sqrt{2}}, \quad (5)$$

Here f_{abc} are the group structure constants and the internal space volume is $\Omega = \int |U_m^a| d^6 \mathbf{z}$. In the maximal case, i. e., $SU(2) \times SU(2)$, each S^3 factor admits invariant 1-form $\theta^a = \theta_i^a dz^i$, which satisfies

$$d\theta^a + \frac{1}{2} \epsilon_{abc} \theta^b \wedge \theta^c = 0. \quad (6)$$

If one chooses

$$U_m^a \equiv U_i^a = -\frac{\sqrt{2}}{g} \theta_i^a, \quad (7)$$

where g is a coupling constant, then the structure constants will be given in terms of the coupling constant by $f_{abc} = g\epsilon_{abc}$. For the case where the coupling constant of one of the $SU(2)$ factors vanishes, the internal space becomes the group manifold $SU(2) \times [U(1)]^3$.

Our ansatz for the reduction to five dimensions is given by the following parameterization of the vielbein

$$\hat{e}_M^A = \begin{pmatrix} e^{\frac{5}{6}\hat{\phi}} e_\mu^\alpha & \sqrt{2} A_\mu^a e^{-\frac{1}{2}\hat{\phi}} \\ 0 & e^{-\frac{1}{2}\hat{\phi}} U_m^a \end{pmatrix}. \quad (8)$$

Here the function U depends only on the internal coordinates $(5, \dots, 9)$. As an internal space we take the group manifold $SU(2) \times [U(1)]^2$, this means that the following choice for the structure constants is taken,

$$\begin{aligned} f_{mnp} &= g\epsilon_{mnp}, & m, n, p &= 5, 6, 7, \\ f_{mnp} &= 0, & m, n, p &= 8, 9 \end{aligned} \quad (9)$$

The $S_{\hat{G}}$ term of $D = 10$ supergravity gives upon reduction, the following $D = 5$ Lagrangian

$$\mathcal{L}_{5G} = e \left(-\frac{1}{4}R - \frac{1}{8}e^{-\frac{8}{3}\hat{\phi}}F_{\mu\nu}^a F^{\mu\nu a} + \frac{5}{6}g^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi} + \frac{3g^2}{16}e^{\frac{8}{3}\hat{\phi}} \right) \quad (10)$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f_{abc}A_\mu^b A_\nu^c \quad (11)$$

This is obtained by using the general formula of Scherk and Schwarz [12] for reducing gravity from higher dimensions. For the antisymmetric tensor, we take the ansatz

$$\hat{B}_{\mu\nu} = B_{\mu\nu}, \quad \hat{B}_{\mu m} = -\frac{1}{\sqrt{2}}A_\mu^a U_m^a(z), \quad \hat{B}_{mn} = \tilde{B}_{mn}(z) \quad (12)$$

such that $\hat{H}_{mnp} = \frac{1}{2\sqrt{2}}f_{mnp}$. To evaluate $\hat{H}_{MNP}\hat{H}^{MNP}$ we first define

$$\hat{H}_{ABC} = \hat{e}_A^M \hat{e}_B^N \hat{e}_C^P \hat{H}_{MNP}.$$

A direct substitution of the ansatz (8) and (12) gives

$$\begin{aligned} \hat{H}_{\alpha\beta\gamma} &= e^{-\frac{5}{2}\hat{\phi}}e_\alpha^\mu e_\beta^\nu e_\gamma^\rho H'_{\mu\nu\rho}, \\ \hat{H}_{\alpha\beta c} &= \frac{1}{\sqrt{2}}e^{-\frac{7}{6}\hat{\phi}}e_\alpha^\mu e_\beta^\nu F_{\mu\nu}^a, \\ \hat{H}_{\alpha bc} &= 0, \\ \hat{H}_{abc} &= \frac{1}{2\sqrt{2}}f_{abc} \end{aligned}$$

where

$$\begin{aligned} H'_{\mu\nu\rho} &= H_{\mu\nu\rho} - \omega_{\mu\nu\rho}, \\ \omega_{\mu\nu\rho} &= -6(A_{[\mu}^a \partial_\nu A_{\rho]}^a + \frac{1}{3}f_{abc}A_\mu^a A_\nu^b A_\rho^c). \end{aligned} \quad (13)$$

These results are a very strong consistency checks on the ansatz, especially in the form of $H'_{\mu\nu\rho}$ and $\hat{H}_{\alpha\beta c}$. We now have

$$\mathcal{L}_{5B} = e \left(\frac{1}{12} e^{-\frac{16}{3}\hat{\phi}} H'_{\mu\nu\rho} H'^{\mu\nu\rho} - \frac{1}{8} e^{-\frac{8}{3}\hat{\phi}} F_{\mu\nu}^a F^{\mu\nu a} - \frac{g^2}{16} e^{\frac{8}{3}\hat{\phi}} \right). \quad (14)$$

The scalar part of $D = 10$ supergravity, gives the following contribution to the five dimensional theory

$$\mathcal{L}_{5S} = \frac{e}{2} \partial_\mu \hat{\phi} \partial^\mu \hat{\phi}. \quad (15)$$

Therefore, combining all terms, the five dimensional theory is described by the Lagrangian

$$\mathcal{L}_5 = e \left(-\frac{1}{4} R - \frac{1}{4} e^{-\frac{8}{3}\hat{\phi}} F_{\mu\nu}^a F^{\mu\nu a} + \frac{4}{3} g^{\mu\nu} \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} + \frac{1}{12} e^{-\frac{16}{3}\hat{\phi}} H'_{\mu\nu\rho} H'^{\mu\nu\rho} + \frac{g^2}{8} e^{\frac{8}{3}\hat{\phi}} \right). \quad (16)$$

Since the potential of the resulting theory depends only on one scalar field, a consistent truncation can be achieved by setting all gauge fields but one to zero. Therefore the index a in the above Lagrangian will take one value. In order to compare with the standard Lagrangian, we multiply our Lagrangian by a factor of 2. Therefore the resulting $N = 2$ five dimensional theory is described by the Lagrangian

$$\mathcal{L}_5 = e \left(-\frac{1}{2} R - \frac{1}{2} e^{-\frac{8}{3}\hat{\phi}} F_{\mu\nu}^1 F^{\mu\nu 1} + \frac{8}{3} g^{\mu\nu} \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} + \frac{1}{6} e^{-\frac{16}{3}\hat{\phi}} H'_{\mu\nu\rho} H'^{\mu\nu\rho} + \frac{g^2}{4} e^{\frac{8}{3}\hat{\phi}} \right), \quad (17)$$

where $\omega_{\mu\nu\rho} = -6A_{[\mu}^1 \partial_\nu A_{\rho]}^1$ in $H'_{\mu\nu\rho}$. We now apply a duality transformation by adding to the above Lagrangian the term

$$-\frac{1}{3} \epsilon^{\mu\nu\rho\sigma\kappa} \partial_\mu A_\nu^0 H_{\rho\sigma\kappa}.$$

Integrating A_ν^0 forces $H_{\mu\nu\rho}$ to be of the form $3\partial_{[\mu} B_{\nu\rho]}$ giving the five dimensional theory with the $B_{\mu\nu}$ field. On the other hand by integrating the independent field $H_{\mu\nu\rho}$ in the path integral as it appears linearly and quadratically is equivalent to the substitution

$$H_{\mu\nu\rho} = \omega_{\mu\nu\rho} + e^{\frac{16}{3}\hat{\phi}} \epsilon_{\mu\nu\rho}^{\sigma\kappa} \partial_\sigma A_\kappa^0.$$

This gives the dual Lagrangian

$$\begin{aligned} \mathcal{L}_5 = & e \left(-\frac{1}{2}R - \frac{1}{2}e^{-\frac{8\hat{\phi}}{3}}F_{\mu\nu}^1F^{\mu\nu 1} - \frac{1}{2}e^{\frac{16\hat{\phi}}{3}}F_{\mu\nu}^0F^{\mu\nu 0} + \frac{8}{3}g^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi} \right. \\ & \left. + \frac{e^{-1}}{2}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}^1F_{\rho\sigma}^1A_\lambda^0 + \frac{g^2}{4}e^{\frac{8\hat{\phi}}{3}} \right). \end{aligned} \quad (18)$$

where 0 and 1 are to label the graviphoton and the additional vector multiplet gauge fields respectively. This is the gauged $U(1)$ five dimensional $N = 2$ supergravity with one vector multiplet. It is the gauged version of the five dimensional theory initially introduced by Strominger and Vafa [11].

We close this section by noting that it is possible to obtain $D = 5$ gauged supergravity from $D = 7$ gauged supergravity. In a recent work we have derived $D = 7$ gauged supergravity by compactifying $D = 10$ on an $SU(2)$ group manifold [6]. The Lagrangian in seven dimensions is given by

$$\mathcal{L}_7 = e \left(-\frac{1}{4}R - \frac{1}{4}e^{-\frac{8}{5}\hat{\phi}}F_{MN}^aF^{MN a} + \frac{4}{5}g^{MN}\partial_M\hat{\phi}\partial_N\hat{\phi} + \frac{1}{12}e^{-\frac{16}{5}\hat{\phi}}H'_{MNP}H'^{MNP} + \frac{g^2}{8}e^{\frac{8}{5}\hat{\phi}} \right).$$

This is reduced and truncated to $D = 5$ gauged supergravity by taking the following ansatz

$$e_M^A = \begin{pmatrix} e^{\frac{8}{15}\hat{\phi}}e_\mu^\alpha & 0 \\ 0 & e^{-\frac{4}{5}\hat{\phi}}\delta_m^i \end{pmatrix}, \quad m = 5, 6.$$

as well as $B_{\mu m} = 0$ and $A_m^a = 0$. One can easily show that the reduced Lagrangian is given by (16)

3 Embedding Into Very Special Geometry

The solutions of five dimensional $N = 2$ supergravity theory with vector multiplets theory have been discussed within the framework of very special geometry [7, 8, 9, 10]. Therefore, before we discuss solutions of the five dimensional theory and their embedding into ten dimensional supergravity and M-theory, it is essential to consider our compactified Lagrangian in this framework.

A class of five-dimensional $N = 2$ supergravity coupled to abelian vector supermultiplets can be obtained by compactifying eleven-dimensional supergravity, the low-energy theory of M-theory, on a Calabi-Yau three-folds [13]. The massless spectrum of the theory contains $(h_{(1,1)} - 1)$ vector multiplets with real scalar components, and thus $h_{(1,1)}$ vector bosons (the additional vector boson is the graviphoton). The theory also contains $h_{(2,1)} + 1$ hypermultiplets, where $h_{(1,1)}$ and $h_{(2,1)}$, are the Calabi-Yau Hodge numbers.

The bosonic part of the effective gauged supersymmetric $N = 2$ Lagrangian which describes the coupling of vector multiplets to supergravity is given by [2]

$$\mathcal{L} = e \left(\frac{1}{2} R + g^2 V - \frac{1}{4} G_{IJ} F_{\mu\nu}^I F^{\mu\nu J} - \frac{1}{2} \mathcal{G}_{ij} \partial_\mu \phi^i \partial^\mu \phi^j + \frac{e^{-1}}{48} \epsilon^{\mu\nu\rho\sigma\lambda} C_{IJK} F_{\mu\nu}^I F_{\rho\sigma}^J A_\lambda^K \right), \quad (19)$$

R is the scalar curvature, $F_{\mu\nu}^I$ are the Abelian field-strength tensor, V is the potential given by

$$V(X) = V_I V_J \left(6 X^I X^J - \frac{9}{2} \mathcal{G}^{ij} \partial_i X^I \partial_j X^J \right), \quad (20)$$

where X^I represent the real scalar fields which have to satisfy the constraint

$$\mathcal{V} = \frac{1}{6} C_{IJK} X^I X^J X^K = 1. \quad (21)$$

Also:

$$G_{IJ} = -\frac{1}{2} \partial_I \partial_J \log \mathcal{V} \Big|_{\mathcal{V}=1}, \quad \mathcal{G}_{ij} = \partial_i X^I \partial_j X^J G_{IJ} \Big|_{\mathcal{V}=1}, \quad (22)$$

where ∂_i refers to a partial derivative with respect to the scalar field ϕ^i . The physical quantities in (19) can all be expressed in terms of the homogeneous cubic polynomial \mathcal{V} .

Further useful relations are

$$\partial_i X_I = -\frac{2}{3} G_{IJ} \partial_i X^J, \quad X_I = \frac{2}{3} G_{IJ} X^J. \quad (23)$$

It is worth pointing out that for Calabi-Yau compactification, \mathcal{V} is the intersection form, X^I and $X_I = \frac{1}{6} C_{IJK} X^J X^K$ correspond to the size of the two- and four-cycles and C_{IJK} are the intersection numbers of the Calabi-Yau threefold.

A very useful relation of very special geometry is

$$\mathcal{G}^{ij}\partial_j X^I \partial_j X^J = G^{IJ} - \frac{2}{3}X^I X^J. \quad (24)$$

The potential can also be written as

$$V(X) = 9 V_I V_J \left(X^I X^J - \frac{1}{2} G^{IJ} \right). \quad (25)$$

The Lagrangian (18) correspond to the following identifications²

$$G_{00} = 2e^{\frac{16\hat{\phi}}{3}}, \quad G_{11} = 2e^{-8\frac{\hat{\phi}}{3}}, \quad \mathcal{G}_{11} = \frac{16}{3}, \quad C_{011} = 8.$$

In order to determine the X^I , we use the relations (23) together with $X_I X^I = 1$, this gives $X^0 = c_1 e^{-8\frac{\hat{\phi}}{3}}$, $X^1 = c_2 e^{4\frac{\hat{\phi}}{3}}$. Upon using (24) and the above identification we get $c_1 = \frac{1}{2}$, $c_2 = \frac{1}{\sqrt{2}}$. Finally, from the expression of the potential one obtains that $V_0 = 0$, $V_1 = \frac{1}{3}$. Therefore we have

$$X^0 = \frac{1}{2}e^{-8\frac{\hat{\phi}}{3}}, \quad X^1 = \frac{1}{\sqrt{2}}e^{4\frac{\hat{\phi}}{3}}, \quad X_0 = \frac{2}{3}e^{8\frac{\hat{\phi}}{3}}, \quad X_1 = \frac{2\sqrt{2}}{3}e^{-4\frac{\hat{\phi}}{3}}.$$

4 Lifting Solutions to ten and eleven dimensions

Our previous results suggest that any solution of gauged supergravity in seven and five dimension given in terms of the metric, gauge field and scalar fields can be lifted to ten dimensions as a solution of $N = 1$ ten dimensional supergravity. By also noting the relation between the ten and eleven dimensional theory, one can then lift all solutions to eleven dimensions.

To lift the solutions to ten dimensions we have to express the ten dimensional fields in terms of five dimensional ones. To start with we write the ten dimensional metric as

$$\begin{aligned} \hat{g}_{\mu\nu} &= e^{\frac{5}{3}\hat{\phi}} g_{\mu\nu} - 2e^{-\hat{\phi}} A_\mu^a A_\nu^a, \\ \hat{g}_{\mu m} &= -\sqrt{2}e^{-\hat{\phi}} A_\mu^a U_m^a(z), \\ \hat{g}_{mn} &= -e^{-\hat{\phi}} U_m^a(z) U_n^a(z). \end{aligned}$$

²note that the sign difference of the kinetic terms is due to using a different metric signature

The five dimensional model is given in terms of X^0, X^1, A_μ^0 and A_μ^1 . Since A_μ^0 is the dual of $B_{\mu\nu}$ we first evaluate

$$H'_{\mu\nu\rho} = H_{\mu\nu\rho} - \omega_{\mu\nu\rho} = e^{\frac{16}{3}\hat{\phi}} \epsilon_{\mu\nu\rho}^{\sigma\kappa} \partial_\sigma A_\kappa^0.$$

Or in terms of the ten dimensional fields

$$\begin{aligned} \hat{H}_{\alpha\beta\gamma} &= e^{-\frac{5}{2}\hat{\phi}} e_\alpha^\mu e_\beta^\nu e_\gamma^\rho H'_{\mu\nu\rho} = e^{\frac{17}{6}\hat{\phi}} \epsilon_{\alpha\beta\gamma}^{\delta\eta} e_\delta^\sigma e_\eta^\kappa \partial_\sigma A_\kappa^0, \\ \hat{H}_{\alpha\beta c} &= \frac{1}{\sqrt{2}} \delta_c^1 e^{-\frac{7}{6}\hat{\phi}} e_\alpha^\mu e_\beta^\nu F_{\mu\nu}^1. \end{aligned}$$

The dilaton field is determined from the solution to X^0 and X^1 .

4.1 Electrically charged solutions

The spherically symmetric BPS electric solutions as well as magnetic string solutions were obtained in [7, 9] by solving for the vanishing of the gravitino and gaugino supersymmetry variation for a particular choice of the supersymmetry parameter. These are given by

$$\begin{aligned} ds^2 &= -\mathcal{V}^{-4/3} (1 + g^2 r^2 \mathcal{V}^2) dt^2 + \mathcal{V}^{2/3} \left[\frac{dr^2}{1 + g^2 r^2 \mathcal{V}^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2) \right] \\ F_{tm}^I &= -\partial_m (\mathcal{V}^{-1} Y^I) \quad , \\ \mathcal{V} &= \frac{1}{6} C_{IJK} Y^I Y^J Y^K, \quad \frac{1}{2} C_{IJK} Y^J Y^K = H_I = 3V_I + \frac{q_I}{r^2} \end{aligned} \tag{26}$$

where

$$Y^I = \mathcal{V}^{\frac{2}{3}} X^I, \quad Y_I = \mathcal{V}^{\frac{1}{3}} X_I, \quad \mathcal{V} = e^{3U}$$

Let us go back to the solutions of our five dimensional gauged theory with one vector multiplet. As for the electric solutions, we have the following equations

$$\begin{aligned} \mathcal{V} &= \frac{1}{2} C_{011} Y^0 Y^1 Y^1 = 4Y^0 (Y^1)^2, \\ \frac{1}{2} C_{011} (Y^1)^2 &= 4(Y^1)^2 = H_0 = \frac{q_0}{r^2}, \\ C_{110} Y^1 Y^0 &= 8(Y^1) Y^0 = H_1 = 1 + \frac{q_1}{r^2}. \end{aligned} \tag{27}$$

This gives

$$\begin{aligned} Y^0 &= \frac{r}{4q_0^{\frac{1}{2}}} \left(1 + \frac{q_1}{r^2}\right), \\ Y^1 &= \frac{q_0^{\frac{1}{2}}}{2r}, \\ \mathcal{V} &= \frac{q_0^{\frac{1}{2}}}{4r} \left(1 + \frac{q_1}{r^2}\right). \end{aligned}$$

The metric depends only on \mathcal{V} . The gauge field strengths are given by

$$\begin{aligned} F_{tr}^0 &= -\partial_r(\mathcal{V}^{-1}Y^0) = -\frac{2r}{q_0}, \\ F_{tr}^1 &= -\partial_r(\mathcal{V}^{-1}Y^1) = -\frac{4q_1}{r^3 \left(1 + \frac{q_1}{r^2}\right)^2}. \end{aligned} \tag{28}$$

To lift our solution to ten dimensions first we write

$$Y^0 = \mathcal{V}^{\frac{1}{3}} X^0, \quad Y^1 = \mathcal{V}^{\frac{1}{3}} X^1,$$

from which we deduce that

$$\frac{X^0}{X^1} = \frac{Y^0}{Y^1} = \frac{1}{\sqrt{2}} e^{-4\hat{\phi}} = \frac{r^2}{2q_0} \left(1 + \frac{q_1}{r^2}\right).$$

The gauge fields are

$$A_t^0 = \frac{r^2}{q_0}, \quad A_t^1 = \frac{2}{1 + \frac{q_1}{r^2}}.$$

The ten dimensional metric is then

$$\begin{aligned} d\hat{s}^2 &= e^{\frac{5}{3}\hat{\phi}} \left(-\mathcal{V}^{-4/3} (1 + g^2 r^2 \mathcal{V}^2) dt^2 + \mathcal{V}^{2/3} \left[\frac{dr^2}{1 + g^2 r^2 \mathcal{V}^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2) \right] \right) \\ &\quad + e^{-\hat{\phi}} \left(\frac{q_0}{2r^2 \mathcal{V}^2} dt^2 \right) + e^{-\hat{\phi}} \left(\frac{\sqrt{2q_0}}{r\mathcal{V}} U_m^1(z) dt dz^m \right) + e^{-\hat{\phi}} U_m^a(z) U_n^a(z) dz^m dz^n, \end{aligned}$$

where $\mathcal{V} = \frac{q_0^{\frac{1}{2}}}{4r} \left(1 + \frac{q_1}{r^2}\right)$. The non vanishing components of the field strength of the antisymmetric tensor B_{MN} are

$$\begin{aligned}\hat{H}_{234} &= e^{\frac{17}{6}\hat{\phi}} e_0^t e_1^r \partial_r A_t^0 = \frac{q_0^{\frac{7}{8}}}{2^{\frac{5}{16}} r^{\frac{7}{4}} \left(1 + \frac{q_1}{r^2}\right)^{\frac{3}{8}}}, \\ \hat{H}_{015} &= -\frac{1}{\sqrt{2}} e^{-\frac{7}{6}\hat{\phi}} e_0^t e_1^r \partial_r A_t^1 = -\frac{2^{\frac{11}{16}} q_1}{q_0^{\frac{1}{8}} r^{\frac{11}{4}} \left(1 + \frac{q_1}{r^2}\right)^{\frac{11}{8}}}, \\ \hat{H}_{567} &= \frac{g}{2\sqrt{2}}.\end{aligned}$$

4.2 Magnetic solutions

Here we will discuss the magnetic string solution found in [9]. This is given by the metric

$$\begin{aligned}ds^2 &= (gr)^{\frac{1}{2}} e^{-\frac{3U}{2}} (-dt^2 + dz^2) + e^{2U} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \\ e^{-U} &= \frac{1}{3gr} + gr.\end{aligned}\tag{29}$$

The gauge fields are given by $A_\phi^I = -q^I \cos \theta$ and the scalar fields and the magnetic charges satisfy

$$X^I V_I = 1, \quad 3gq^I V_I = 1.\tag{30}$$

As for the magnetic solution of our model one finds that

$$X^0 = \frac{1}{36}, \quad X_0 = 12, \quad X^1 = 3, \quad X_1 = \frac{2}{9}, \quad e^{\frac{4\hat{\phi}}{3}} = 3\sqrt{2}.\tag{31}$$

The above solution can be lifted to ten dimensions and we get

$$\begin{aligned}d\hat{s}^2 &= (18)^{\frac{5}{8}} \left((gr)^{\frac{1}{2}} e^{-\frac{3U}{2}} (-dt^2 + dz^2) + e^{2U} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right) \\ &\quad + 2(18)^{-\frac{1}{8}} q_1^2 \cos^2 \theta d\phi^2 - 2^{\frac{3}{4}} (18)^{-\frac{1}{8}} q_1 \cos \theta U_m^1(z) d\phi dz^m \\ &\quad + (18)^{-\frac{1}{8}} U_m^a(z) U_n^a(z) dz^m dz^n.\end{aligned}$$

$$\hat{H}_{012} = e^{\frac{17}{6}\hat{\phi}} e_3^\theta e_4^\phi \partial_\theta A_\phi^0 = 18 \frac{17}{48} \frac{q_0}{r^2}.$$

We note that the numerical factors for the solution can be absorbed by rescaling the coordinates and the charge q_1 . More solutions found in [10] can also be lifted.

To lift these solutions further to eleven dimensions we have to first write the dimensional reduction of eleven dimensional supergravity to $N = 1$ ten dimensional supergravity. These are [14]

$$E_M^A = e^{-\frac{1}{6}\hat{\phi}} \hat{e}_M^A, \quad E_{11}^{11} = e^{\frac{4}{3}\hat{\phi}}, \quad A_{MN11} = \hat{B}_{MN}$$

and the eleven dimensional metric is related to the ten dimensional one by

$$ds^{2(eleven)} = e^{-\frac{1}{3}\hat{\phi}} d\hat{s}^2 + e^{\frac{8}{3}\hat{\phi}} (dx^{11})^2.$$

The non vanishing components of the antisymmetric tensor field-strengths are

$$F_{MNP11} = \hat{H}_{MNP}.$$

The lifting of solutions from five to seven dimensions is very simple. We write

$$ds_7^2 = e^{\frac{16}{15}\hat{\phi}} ds_5^2 + e^{-\frac{8}{5}\hat{\phi}} \left((dx^5)^2 + (dx^6)^2 \right).$$

5 Conclusions

In this work we have shown that it is possible to obtain $U(1)$ gauged $N = 2$ five dimensional supergravity interacting with one vector multiplet by compactifying and truncating ten dimensional supergravity on the group manifold $SU(2) \times U(1)^2$. The model obtained is the gauged version of the supergravity model introduced by Strominger and Vafa. Using the relation between the higher dimensional fields and the lower ones, it becomes possible to lift known solutions such as black holes, string solutions and domain walls of the five dimensional theory to seven, ten and eleven dimensional supergravity theories to ten and eleven dimensions. Some known electric and magnetic solutions for our gauged $D = 5$ supergravity compactified model, formulated in the framework of special geometry, were lifted to higher dimensions. Such solutions are not easy to find directly by studying the seven,

ten and eleven dimensional supergravity theories. At this stage it would be useful to study some of the properties of these solutions and give their interpretation in terms of D-brane and M-theory dynamics.

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